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The evolution of clustering and bias in the galaxy distribution

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This paper reviews the measurements of galaxy correlations at high redshifts, and discusses how these may be understood in models of hierarchical gravitational collapse. The clustering of galaxies at redshift one is much weaker than at present, and this is consistent with the rate of growth of structure expected in an open universe. If $\Omega = 1$, this observation would imply that bias increases at high redshift, in conflict with observed M/L values for known high- z clusters. At redshift 3, the population of Lyman-limit galaxies displays clustering that is of similar amplitude to that seen today. This is most naturally understood if the Lyman-limit population is a set of rare recently formed objects. Knowing both the clustering and the abundance of these objects it is possible to deduce, empirically, the fluctuation spectrum required on scales that cannot be measured today owing to gravitational nonlinearities. Of existing physical models for the fluctuation spectrum, the results are most closely matched by a low-density spatially flat universe. This conclusion is reinforced by an empirical analysis of CMB anisotropies, in which the present-day fluctuation spectrum is forced to have the observed form. Open models are strongly disfavoured, leaving Λ CDM as the most successful simple model for structure formation.

Keywords: galaxy clustering; correlation functions; galaxy formation; microwave background; high-redshift galaxies

1. Background

(a) *Evolution of mass fluctuations*

Attempts to understand the evolution of structure in the galaxy distribution start with the assumption that this evolution is directly related to gravitationally driven evolution of the dark matter. This is a well-understood problem, with the following features.

1. *Linear evolution.* The fractional density contrast δ evolves according to linear perturbation theory as

$$\delta \propto a(t)g(\Omega); \quad g(\Omega) \simeq \begin{cases} \Omega^{0.65} & \text{open,} \\ \Omega^{0.23} & \text{flat,} \end{cases} \quad (1.1)$$

where $a(t) = (1+z)^{-1}$ is the scale factor, and the growth suppression factor, $g(\Omega)$, is much less important for $k = 0$ models; the universe only discovers rather late that there is a non-zero Λ (Lahav *et al.* 1991; Carroll *et al.* 1992).

2. *Stable clustering.* In the opposite extreme of highly nonlinear clustering, there is Peebles's concept of stable clustering, in which virialized objects maintain a

fixed proper size and merely change their separation with time. This leads to the common parametrization for the correlation function in co-moving coordinates:

$$\xi(r, z) = [r/r_0]^{-\gamma}(1+z)^{-(3-\gamma+\epsilon)}, \quad (1.2)$$

where $\epsilon = 0$ is stable clustering; $\epsilon = \gamma - 3$ is constant co-moving clustering; $\epsilon = \gamma - 1$ is $\Omega = 1$ linear-theory evolution.

Although this equation is frequently encountered, it is probably not applicable to the real world, because most data inhabit the intermediate regime of $1 \lesssim \xi \lesssim 100$. Peacock (1997) showed that the expected evolution in this quasi-linear regime is significantly more rapid: up to $\epsilon \simeq 3$.

(b) *General aspects of bias*

Of course, there are good reasons to expect that the galaxy distribution will not follow that of the dark matter. The main empirical argument in this direction comes from the masses of rich clusters of galaxies. It has long been known that attempts to ‘weigh’ the universe by multiplying the overall luminosity density by cluster M/L ratios give apparent density parameters in the range $\Omega \simeq 0.2$ – 0.3 (see, for example, Carlberg *et al.* 1996).

An alternative argument is to use the abundance of rich clusters of galaxies in order to infer the RMS fractional density contrast in spheres of radius $8 h^{-1}$ Mpc. This calculation has been carried out in several different ways, with general agreement on a figure close to

$$\sigma_8 \simeq 0.57 \Omega_m^{-0.56} \quad (1.3)$$

(White *et al.* 1993; Eke *et al.* 1996; Viana & Liddle 1996). The observed apparent value of σ_8 in, for example, APM galaxies (Maddox *et al.* 1996) is about 0.95 (ignoring nonlinear corrections, which are small in practice, although this is not obvious in advance). This says that $\Omega = 1$ needs substantial positive bias, but that $\Omega \lesssim 0.4$ needs *anti*-bias. Although this cluster normalization argument depends on the assumption that the density field obeys Gaussian statistics, the required degree of bias is in reasonable agreement with what is inferred from cluster M/L ratios.

What effect does bias have on common statistical measures of clustering such as correlation functions? We could be perverse and assume that the mass and light fields are completely unrelated. If, however, we are prepared to make the more sensible assumption that the light density is a nonlinear but local function of the mass density, then there is a very nice result due to Coles (1993): the bias is a monotonic function of scale. Explicitly, if scale-dependent bias is defined as

$$b(r) \equiv [\xi_{\text{galaxy}}(r)/\xi_{\text{mass}}(r)]^{1/2}, \quad (1.4)$$

then $b(r)$ varies monotonically with scale under rather general assumptions about the density field. Furthermore, at large r , the bias will tend to a constant value, which is the linear response of the galaxy-formation process.

There is certainly empirical evidence that bias in the real universe does work this way. Consider figure 1, taken from Peacock (1997). This compares dimensionless power spectra ($\Delta^2(k) = d\sigma^2/d \ln k$) for IRAS and APM galaxies. The comparison is made in real space, to avoid distortions due to peculiar velocities. For IRAS galaxies,

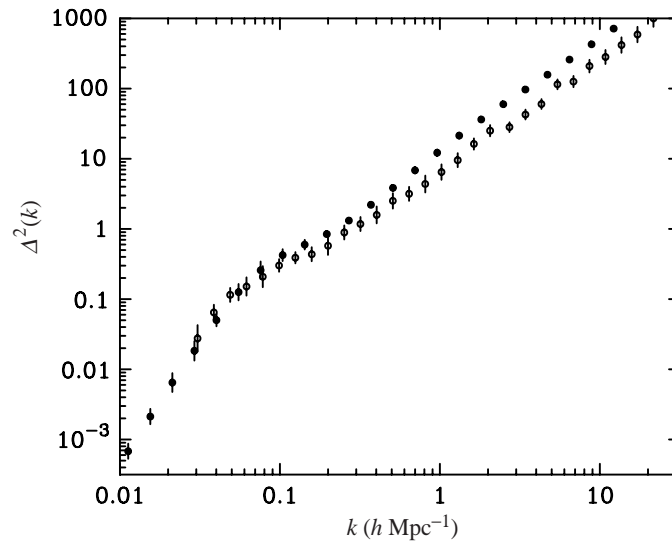


Figure 1. The real-space power spectra of optically selected APM galaxies (solid circles) and IRAS galaxies (open circles), taken from Peacock (1997). IRAS galaxies show weaker clustering, consistent with their suppression in high-density regions relative to optical galaxies. The relative bias is a monotonic but slowly varying function of scale.

the real-space power was obtained from the projected correlation function:

$$\Xi(r) = \int_{-\infty}^{\infty} \xi[(r^2 + x^2)^{1/2}] dx. \quad (1.5)$$

Saunders *et al.* (1992) describe how this statistic can be converted to other measures of real-space correlation. For the APM galaxies, Baugh & Efstathiou (1993, 1994) deprojected Limber's equation for the angular correlation function $w(\theta)$ (discussed below). These different methods yield rather similar power spectra, with a relative bias that is perhaps only about 1.2 on large scale, increasing to about 1.5 on small scales. The power-law portion for $k \gtrsim 0.2 h \text{ Mpc}^{-1}$ is the clear signature of nonlinear gravitational evolution, and the slow scale dependence of bias gives encouragement that the galaxy correlations give a good measure of the shape of the underlying mass-fluctuation spectrum.

2. Observations of high-redshift clustering

(a) Clustering at redshift 1

At $z = 0$, there is a degeneracy between Ω and the true normalization of the spectrum. Since the evolution of clustering with redshift depends on Ω , studies at higher redshifts should be capable of breaking this degeneracy. This can be done without using a complete faint redshift survey, by using the angular clustering of a flux-limited survey. If the form of the redshift distribution is known, the projection effects can be disentangled in order to estimate the three-dimensional clustering at the average redshift of the sample. For small angles, and where the redshift shell being studied is thicker than the scale of any clustering, the spatial and angular correlation

functions are related by Limber's equation (see, for example, Peebles 1980):

$$w(\theta) = \int_0^\infty y^4 \phi^2(y) C(y) dy \int_{-\infty}^\infty \xi([x^2 + y^2 \theta^2]^{1/2}, z) dx, \quad (2.1)$$

where y is a dimensionless co-moving distance (transverse part of the FRW metric is $[R(t)y d\theta]^2$), and $C(y) = [1 - ky^2]^{-1/2}$; the selection function for radius y is normalized so that $\int y^2 \phi(y) C(y) dy = 1$. Less well known, but simpler, is the Fourier analogue of this relation:

$$\Delta_\theta^2(K) = \frac{\pi}{K} \int \Delta^2([K/y], z) y^5 \phi^2(y) C(y) dy, \quad (2.2)$$

where Δ_θ^2 is the angular power spectrum and K is the angular wavenumber (Kaiser 1992). In either case, the angular clustering tends to be sensitive to the spatial clustering at the redshift, \bar{z} , at which $y^2 \phi(y)$ peaks.

This relation has been used by many workers in order to interpret angular clustering of faint galaxies (see, for example, Efstathiou *et al.* 1991; Neuschaefer *et al.* 1991; Couch *et al.* 1993; Roche *et al.* 1993). The general conclusion was always that clustering seemed to be weaker in the past, but the rate of evolution was not very well tied down, owing to uncertainties in the redshift distribution for faint galaxies, plus the fact that projection effects leave only a very small clustering signal. The uncertainties in interpreting $w(\theta)$ for faint galaxies were first convincingly overcome by the CFRS team, who assembled a large enough redshift survey to construct the correlation function directly out to $z \simeq 1$ (Le Fèvre *et al.* 1996). Their results were well described by $r_0 \simeq 2 h^{-1}$ Mpc at $z = 1$, i.e. evolution at about the $\epsilon = 1$ rate. Other groups have found similar results (for example, Carlberg *et al.* 1997), although Carlberg's paper in this issue argues for slightly slower evolution. Although this rate of evolution is in accord with the expected linear-theory evolution in an $\Omega = 1$ model, the discussion of § 1*a* shows that such a result is in fact more consistent with lower-density models. Since the data are in the quasi-linear regime, the expected evolution in a critical-density universe would be much more rapid.

The observed clustering at $z \simeq 1$ is thus larger than would be expected if $\Omega = 1$. There is no difficulty with this, since we shall see below that bias is expected to evolve in the sense of being higher at early times. However, consider the implications for cluster M/L ratios: we have already seen that the observed degree of bias at $z = 0$ must reduce these by about a factor of five in the cores of rich clusters. If the bias at $z = 1$ is significantly greater than today, this trend must continue, so that the apparent ' Ω ' from high- z clusters would be expected to be very small. Conversely, if Ω is low today, the $z = 1$ clustering would be nearly unbiased and we would expect to see the true Ω at that time, which should have evolved to be close to unity. So, this leaves the nice paradox that the way to prove $\Omega = 1$ today is to observe a very small ' Ω ' at $z = 1$, and vice versa.

It has recently become possible to carry out this test, through the detection of massive clusters at redshifts near unity. Many of these have been found through X-ray detections, which almost guarantees a high virial temperature and hence a high mass (e.g. the EMSS sample: Henry *et al.* 1992). The existence of massive clusters at high redshift is a potential problem for high-density models, owing to the more rapid evolution of the mass fluctuations in this case, and it has been claimed that $\Omega = 1$ is ruled out (see, for example, Luppino & Gioia 1995; Henry 1997). However, the M/L argument is more powerful since only a single cluster is required, and a complete survey is

not necessary. Two particularly good candidates at $z \simeq 0.8$ are described by Clowe *et al.* (1998); these are clusters where significant weak gravitational-lensing distortions are seen, allowing a robust determination of the total cluster mass. The mean V -band M/L in these clusters is 230 solar units, which is close to typical values in $z = 0$ clusters. However, the co-moving V -band luminosity density of the universe is higher at early times than at present by about a factor of $(1+z)^{2.5}$ (Lilly *et al.* 1996), so this is equivalent to $M/L \simeq 1000$, implying an apparent ' Ω ' of close to unity. In summary, the known degree of bias today coupled with the moderate evolution in correlation function back to $z = 1$ implies that, for $\Omega = 1$, the galaxy distribution at this time would have to consist very nearly of a 'painted-on' pattern that is not accompanied by significant mass fluctuations. Such a picture cannot be reconciled with the healthy M/L ratios that are observed in real clusters at these redshifts, and this seems to be a strong argument that we do not live in an Einstein–de Sitter universe.

(b) Clustering of Lyman-limit galaxies at redshift 3

The most exciting recent development in observational studies of galaxy clustering is the detection by Steidel *et al.* (1998) of strong clustering in the population of Lyman-limit galaxies at $z \simeq 3$. The evidence takes the form of a redshift histogram binned at $\Delta z = 0.04$ resolution over a field 8.7×17.6 in extent. For $\Omega = 1$ and $z = 3$, this probes the density field using a cell with dimensions

$$\text{cell} = 15.4 \times 7.6 \times 15.0 [h^{-1} \text{Mpc}]^3. \quad (2.3)$$

Conveniently, this has a volume equivalent to a sphere of radius $7.5 h^{-1} \text{Mpc}$, so it is easy to measure the bias directly by reference to the known value of σ_8 . Since the degree of bias is large, redshift-space distortions from coherent infall are small; the cell is also large enough that the distortions of small-scale random velocities at the few hundred km s^{-1} level are also small. Using the model of eqn (11) of Peacock (1997) for the anisotropic redshift-space power spectrum and integrating over the exact anisotropic window function, the above simple volume argument is found to be accurate to a few per cent for reasonable power spectra:

$$\sigma_{\text{cell}} \simeq b(z=3)\sigma_{7.5}(z=3), \quad (2.4)$$

defining the bias factor at this scale. The results of §1 (see also Mo & White 1996) suggest that the scale-dependence of bias should be weak.

In order to estimate σ_{cell} , simulations of synthetic redshift histograms were made, using the method of Poisson-sampled lognormal realizations described by Broadhurst *et al.* (1995): using a χ^2 statistic to quantify the non-uniformity of the redshift histogram, it appears that $\sigma_{\text{cell}} \simeq 0.9$ is required in order for the field of Steidel *et al.* (1998) to be typical. It is then straightforward to obtain the bias parameter since, for a present-day correlation function $\xi(r) \propto r^{-1.8}$,

$$\sigma_{7.5}(z=3) = \sigma_8 \times [8/7.5]^{1.8/2} \times \frac{1}{4} \simeq 0.146, \quad (2.5)$$

implying

$$b(z=3 | \Omega=1) \simeq 0.9/0.146 \simeq 6.2. \quad (2.6)$$

Steidel *et al.* (1998) use a rather different analysis that concentrates on the highest peak alone, and obtain a minimum bias of 6, with a preferred value of 8. They use

the Eke *et al.* (1996) value of $\sigma_8 = 0.52$, which is on the low side of the published range of estimates. Using $\sigma_8 = 0.55$ would lower their preferred b to 7.6. Note that, with both these methods, it is much easier to rule out a low value of b than a high one; given a single field, it is possible that a relatively 'quiet' region of space has been sampled, and that much larger spikes remain to be found elsewhere. A more detailed analysis of several further fields by Adelberger *et al.* (1998) in fact yields a bias figure very close to that given above, so the first field was apparently not unrepresentative.

Having arrived at a figure for bias if $\Omega = 1$, it is easy to translate to other models, since σ_{cell} is observed, independent of cosmology. For low Ω models, the cell volume will increase by a factor of $[S_k^2(r) dr]/[S_k^2(r_1) dr_1]$; comparing with present-day fluctuations on this larger scale will tend to increase the bias. However, for low Ω , two other effects increase the predicted density fluctuation at $z = 3$: the cluster constraint increases the present-day fluctuation by a factor of $\Omega^{-0.56}$, and the growth between redshift 3 and the present will be less than a factor of four. Applying these corrections gives

$$\frac{b(z = 3 | \Omega = 0.3)}{b(z = 3 | \Omega = 1)} = \begin{cases} 0.42 & \text{(open),} \\ 0.60 & \text{(flat),} \end{cases} \quad (2.7)$$

which suggests an approximate scaling as $b \propto \Omega^{0.72}$ (open) or $\Omega^{0.42}$ (flat). The significance of this observation is thus to provide the first convincing proof for the reality of galaxy bias: for $\Omega \simeq 0.3$, bias is not required in the present universe, but we now see that $b > 1$ is needed at $z = 3$ for all reasonable values of Ω .

(c) Clustering of high-redshift AGN

The strength of clustering for Lyman-limit galaxies fits in reasonably well with what is known about clustering of AGN. A co-moving correlation length of $r_0 \simeq 6.5 h^{-1}$ Mpc has been measured for radio-quiet QSOs at $\langle z \rangle \simeq 1.5$ (Shanks & Boyle 1994; Croom & Shanks 1996). This value is much larger than the clustering of optically selected galaxies at $z \simeq 1$, but this may not be unreasonable, since imaging of QSO hosts reveals them to be several- L^* objects, comparable in stellar mass to radio galaxies (see, for example, Dunlop *et al.* 1993; Taylor *et al.* 1996). It is plausible that the clustering of these massive galaxies at $z \simeq 1$ will be enhanced through exactly the same mechanisms that enhance the clustering of Lyman-limit galaxies at $z \simeq 3$. Of course, this does not rule out more complex pictures based on ideas such as close interactions in rich environments being necessary to trigger AGN. However, as emphasized below, the mass and rareness of these objects sets a *minimum* level of bias. It is to be expected that this bias will increase at higher redshifts, and so one would not expect quasar clustering to decline at higher redshifts. Indeed, it has been claimed that ξ either stays constant at the highest redshifts (Andreani & Cristiani 1992; Croom & Shanks 1996), or even increases (Stephens *et al.* 1997).

Radio-source clustering at high redshifts has been detected only in projection. The FIRST survey has measured $w(\theta)$ to high precision for a limit of 1 mJy at 1.4 GHz (Cress *et al.* 1996). Their result detects clustering at separations between 0.02 and 2 deg, and is fitted by a power law:

$$w(\theta) = 0.003[\theta/\text{deg}]^{-1.1}. \quad (2.8)$$

There had been earlier claims of detections of angular clustering, notably the 87GB survey (Loan *et al.* 1997), but these were of only bare significance (although, in retrospect, the level of clustering in 87GB is consistent with the FIRST measurement). Discussion of the 87GB and FIRST results in terms of Limber's equation has tended to focus on values of ϵ in the region of zero. Cress *et al.* (1996) concluded that the $w(\theta)$ results were consistent with the PN91 value of $r_0 \simeq 10 h^{-1}$ Mpc (although they were not very specific about ϵ). Loan *et al.* (1997) measured $w(1^\circ) \simeq 0.005$ for a 5 GHz limit of 50 mJy, and inferred $r_0 \simeq 12 h^{-1}$ Mpc for $\epsilon = 0$, falling to $r_0 \simeq 9 h^{-1}$ Mpc for $\epsilon = -1$.

The reason for this strong degeneracy between r_0 and ϵ is that r_0 parametrizes the $z = 0$ clustering, whereas the observations refer to a typical redshift of around unity. This means that $r_0(z = 1)$ can be inferred quite robustly to be about $7.5 h^{-1}$ Mpc, without much dependence on the rate of evolution. Since the strength of clustering for optical galaxies at $z = 1$ is known to correspond to the much smaller number of $r_0 \simeq 2 h^{-1}$ Mpc (see, for example, Le Fèvre *et al.* 1996), we see that radio galaxies at this redshift have a relative bias parameter of close to 3. The explanation for this high degree of bias is probably similar to that which applies in the case of QSOs: in both cases we are dealing with AGN hosted by rare massive galaxies.

3. Formation and bias of high-redshift galaxies

The challenge now is to ask how these results can be understood in current models for cosmological structure formation. It is widely believed that the sequence of cosmological structure formation was hierarchical, originating in a density power spectrum with increasing fluctuations on small scales. The large-wavelength portion of this spectrum is accessible to observation today through studies of galaxy clustering in the linear and quasi-linear regimes. However, nonlinear evolution has effectively erased any information on the initial spectrum for wavelengths below about 1 Mpc. The most sensitive way of measuring the spectrum on smaller scales is via the abundances of high-redshift objects; the amplitude of fluctuations on scales of individual galaxies governs the redshift at which these objects first undergo gravitational collapse. The small-scale amplitude also influences clustering, since rare early forming objects are strongly correlated, as first realized by Kaiser (1984). It is therefore possible to use observations of the abundances and clustering of high-redshift galaxies to estimate the power spectrum on small scales, and the following section summarizes the results of this exercise, as given by Peacock *et al.* (1998).

(a) Press–Schechter apparatus

The standard framework for interpreting the abundances of high-redshift objects in terms of structure-formation models was outlined by Efstathiou & Rees (1988). The formalism of Press & Schechter (1974) gives a way of calculating the fraction F_c of the mass in the universe which has collapsed into objects more massive than some limit M :

$$F_c(> M, z) = 1 - \operatorname{erf} \left[\frac{\delta_c}{\sqrt{2}\sigma(M)} \right]. \quad (3.1)$$

Here, $\sigma(M)$ is the RMS fractional density contrast obtained by filtering the linear-theory density field on the required scale. In practice, this filtering is usually performed with a spherical 'top hat' filter of radius R , with a corresponding mass of

$\frac{4}{3}\pi\rho_b R^3$, where ρ_b is the background density. The number δ_c is the linear-theory critical overdensity, which for a ‘top-hat’ overdensity undergoing spherical collapse is 1.686; virtually independent of Ω . This form describes numerical simulations very well (see, for example, Ma & Bertschinger 1994). The main assumption is that the density field obeys Gaussian statistics, which is true in most inflationary models. Given some estimate of F_c , the number $\sigma(R)$ can then be inferred. Note that for rare objects this is a pleasingly robust process: a large error in F_c will give only a small error in $\sigma(R)$, because the abundance is exponentially sensitive to σ .

Total masses are, of course, ill-defined, and a better quantity to use is the velocity dispersion. The circular orbital velocity for a halo of mass M and proper radius r is

$$V_c^2 = \frac{GM}{r}. \quad (3.2)$$

For a spherically collapsed object this velocity can be converted directly into a Lagrangian co-moving radius that contains the mass of the object within the virialization radius (see, for example, White *et al.* 1993):

$$R/h^{-1} \text{ Mpc} = \frac{2^{1/2}[V_c/100 \text{ km s}^{-1}]}{\Omega_m^{1/2}(1+z_c)^{1/2}f_c^{1/6}}. \quad (3.3)$$

Here, z_c is the redshift of virialization; Ω_m is the *present* value of the matter density parameter; f_c is the density contrast at virialization of the newly collapsed object relative to the background, which is adequately approximated by

$$f_c = 178/\Omega_m^{0.6}(z_c), \quad (3.4)$$

with only a slight sensitivity to whether Λ is non-zero (Eke *et al.* 1996).

For isothermal-sphere haloes, the velocity dispersion is

$$\sigma_v = V_c/\sqrt{2}. \quad (3.5)$$

Given a formation redshift of interest, and a velocity dispersion, there is then a direct route to the Lagrangian radius from which the proto-object collapsed.

(b) Abundances and masses of high-redshift objects

Three classes of high-redshift object can be used to set constraints on the small-scale power spectrum at high redshift as follows.

(i) Damped Lyman- α systems

Damped Lyman- α absorbers are systems with HI column densities greater than $ca. 2 \times 10^{24} \text{ m}^{-2}$ (Lanzetta *et al.* 1991). If the fraction of baryons in the virialized dark matter haloes equals the global value Ω_B , then data on these systems can be used to infer the total fraction of matter that has collapsed into bound structures at high redshifts (Ma & Bertschinger 1994; Mo & Miralda-Escudé 1994; Kauffmann & Charlot 1994; Klypin *et al.* 1995). The highest measurement at $\langle z \rangle \simeq 3.2$ implies $\Omega_{\text{HI}} \simeq 0.0025h^{-1}$ (Lanzetta *et al.* 1991; Storrie-Lombardi *et al.* 1996). If $\Omega_B h^2 = 0.02$ is adopted, as a compromise between the lower Walker *et al.* (1991) nucleosynthesis estimate and the more recent estimate of 0.025 from Tytler *et al.* (1996), then

$$F_c = \frac{\Omega_{\text{HI}}}{\Omega_B} \simeq 0.12h, \quad (3.6)$$

for these systems. In this case alone, an explicit value of h is required in order to obtain the collapsed fraction; $h = 0.65$ is assumed.

The photoionizing background prevents virialized gaseous systems with circular velocities of less than about 50 km s^{-1} from cooling efficiently, so that they cannot contract to the high-density contrasts characteristic of galaxies (see, for example, Efstathiou 1992). Mo & Miralda-Escudé (1994) used the circular velocity range $50\text{--}100 \text{ km s}^{-1}$ ($\sigma_v = 35\text{--}70 \text{ km s}^{-1}$), to model the damped Lyman- α systems. Reinforcing the photoionization argument, detailed hydrodynamic simulations imply that the absorbers are not expected to be associated with very massive dark-matter haloes (Haehnelt *et al.* 1998). This assumption is consistent with the rather low luminosity galaxies detected in association with the absorbers in a number of cases (Le Brun *et al.* 1996).

(ii) *Lyman-limit galaxies*

Steidel *et al.* (1996) identified star-forming galaxies between $z = 3$ and 3.5 by looking for objects with a spectral break redwards of the U band. The treatment of these Lyman-limit galaxies in this paper is similar to that of Mo & Fukugita (1996), who compared the abundances of these objects to predictions from various models. Steidel *et al.* (1996) give the co-moving density of their galaxies as

$$N(\Omega = 1) \simeq 10^{-2.54} (h^{-1} \text{ Mpc})^{-3}. \quad (3.7)$$

This is a high number density, comparable to that of L^* galaxies in the present universe. The mass of L^* galaxies corresponds to collapse of a Lagrangian region of volume *ca.* 1 Mpc^3 , so the collapsed fraction would be a few tenths of a per cent if the Lyman-limit galaxies had similar masses.

Direct dynamical determinations of these masses are still lacking in most cases. Steidel *et al.* (1996) attempt to infer a velocity width by looking at the equivalent width of the C and Si absorption lines. These are saturated lines, and so the equivalent width is sensitive to the velocity dispersion; values in the range

$$\sigma_v \simeq 180\text{--}320 \text{ km s}^{-1} \quad (3.8)$$

are implied. These numbers may measure velocities that are not due to bound material, in which case they would give an upper limit to $V_c/\sqrt{2}$ for the dark halo. A more recent measurement of the velocity width of the $\text{H}\alpha$ emission line in one of these objects gives a dispersion close to 100 km s^{-1} (M. Pettini, personal communication), consistent with the median velocity width for $\text{Ly}\alpha$ of 140 km s^{-1} measured in similar galaxies in the HDF (Lowenthal *et al.* 1997). Of course, these figures could underestimate the total velocity dispersion, since they are dominated by emission from the central regions only. For the present, the range of values $\sigma_v = 100\text{--}320 \text{ km s}^{-1}$ will be adopted, and the sensitivity to the assumed velocity will be indicated. In practice, this uncertainty in the velocity does not produce an important uncertainty in the conclusions.

(iii) *Red radio galaxies*

An especially interesting set of objects are the reddest optical identifications of 1 mJy radio galaxies, for which deep absorption-line spectroscopy has proved that the red colours result from a well-evolved stellar population, with a minimum stellar

age of 3.5 Gyr for 53W091 at $z = 1.55$ (Dunlop *et al.* 1996; Spinrad *et al.* 1997), and 4.0 Gyr for 53W069 at $z = 1.43$ (Dunlop 1998). Such ages push the formation era for these galaxies back to extremely high redshifts, and it is of interest to ask what level of small-scale power is needed in order to allow this early formation.

Two extremely red galaxies were found at $z = 1.43$ and 1.55, over an area 1.68×10^{-3} sr, so a minimal co-moving density is from one galaxy in this redshift range:

$$N(\Omega = 1) \gtrsim 10^{-5.87} (h^{-1} \text{ Mpc})^{-3}. \quad (3.9)$$

This figure is comparable to the density of the richest Abell clusters, and is thus in reasonable agreement with the discovery that rich high-redshift clusters appear to contain radio-quiet examples of similarly red galaxies (Dickinson 1995).

Since the velocity dispersions of these galaxies are not observed, they must be inferred indirectly. This is possible because of the known present-day Faber–Jackson relation for ellipticals. For 53W091, the large-aperture absolute magnitude is

$$M_V(z = 1.55 | \Omega = 1) \simeq -21.62 - 5 \log_{10} h \quad (3.10)$$

(measured directly in the rest frame). According to solar-metallicity spectral synthesis models, this would be expected to fade by about 0.9 mag. between $z = 1.55$ and the present, for an $\Omega = 1$ model of present age 14 Gyr (note that Bender *et al.* (1996) have observed a shift in the zero-point of the $M-\sigma_v$ relation out to $z = 0.37$ of a consistent size). If we compare these numbers with the σ_v-M_V relation for Coma ($m - M = 34.3$ for $h = 1$) taken from Dressler (1984), this predicts velocity dispersions in the range,

$$\sigma_v = 222\text{--}292 \text{ km s}^{-1}. \quad (3.11)$$

This is a very reasonable range for a giant elliptical, and it is adopted in the following analysis.

Having established an abundance and an equivalent circular velocity for these galaxies, the treatment of them will differ in one critical way from the Lyman- α and Lyman-limit galaxies. For these, the normal Press–Schechter approach assumes the systems under study to be newly born. For the Lyman- α and Lyman-limit galaxies, this may not be a bad approximation, since they are evolving rapidly and/or display high levels of star-formation activity. For the radio galaxies, conversely, their inactivity suggests that they may have existed as discrete systems at redshifts much higher than $z \simeq 1.5$. The strategy will therefore be to apply the Press–Schechter machinery at some unknown formation redshift, and see what range of redshift gives a consistent degree of inhomogeneity.

4. The small-scale fluctuation spectrum

(a) *The empirical spectrum*

Figure 2 shows the $\sigma(R)$ data which result from the Press–Schechter analysis, for three cosmologies. The $\sigma(R)$ numbers measured at various high redshifts have been translated to $z = 0$ using the appropriate linear growth law for density perturbations.

The open symbols give the results for the Lyman-limit (largest- R) and Lyman- α (smallest- R) systems. The approximately horizontal error bars show the effect of the quoted range of velocity dispersions for a fixed abundance; the vertical errors show the effect of changing the abundance by a factor of two at fixed velocity dispersion.

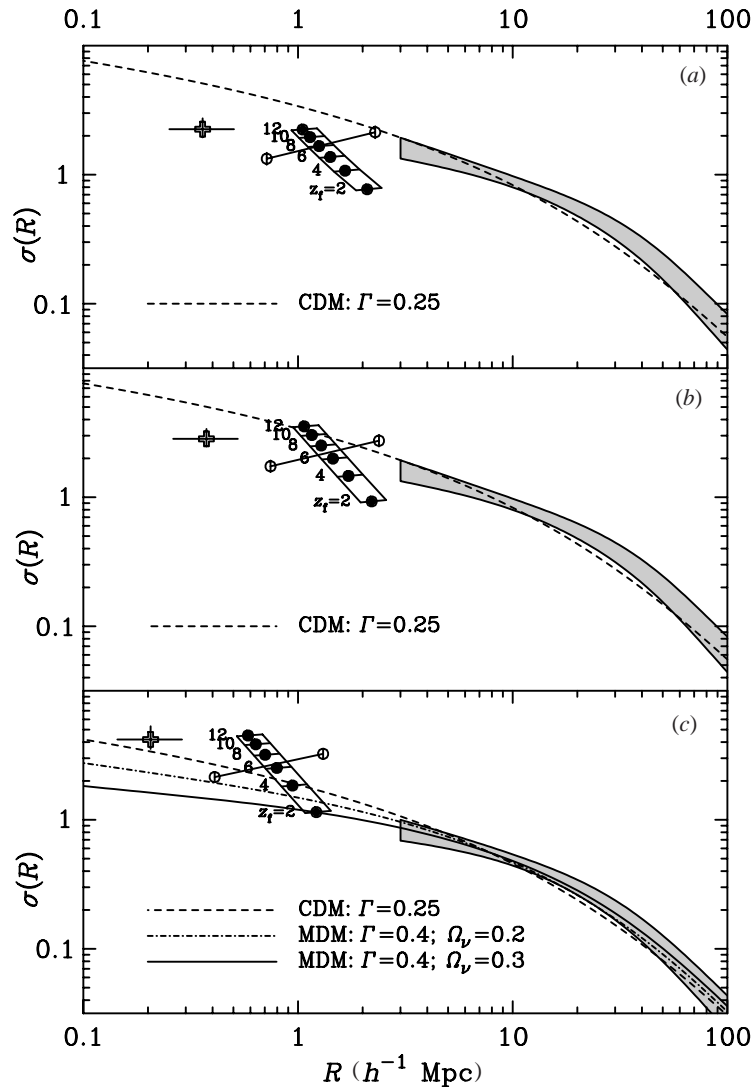


Figure 2. The present-day linear fractional RMS fluctuation in density averaged in spheres of radius R . The data points are Lyman- α galaxies (open cross) and Lyman-limit galaxies (open circles). The diagonal band with solid points shows red radio galaxies with assumed collapse redshifts 2, 4, ..., 12. The vertical error bars show the effect of a change in abundance by a factor of two. The horizontal errors correspond to different choices for the circular velocities of the dark-matter haloes that host the galaxies. The shaded region at large R gives the results inferred from galaxy clustering. The lines show CDM and MDM predictions, with a large-scale normalization of $\sigma_8 = 0.55$ for $\Omega = 1$ or $\sigma_8 = 1$ for the low-density models. (a) $\Omega = 0.3$ open; (b) $\Omega = 0.3$ flat; (c) $\Omega = 1$.

The locus implied by the red radio galaxies sits in between. The different points show the effects of varying collapse redshift: $z_c = 2, 4, \dots, 12$ (lowest redshift gives lowest $\sigma(R)$). Clearly, collapse redshifts of 6–8 are favoured for consistency with the other data on high-redshift galaxies, independent of theoretical preconceptions

and independent of the age of these galaxies. This level of power ($\sigma[R] \simeq 2$ for $R \simeq 1 h^{-1}$ Mpc) is also in very close agreement with the level of power required to produce the observed structure in the Lyman- α forest (Croft *et al.* 1998), so there is a good case to be made that the fluctuation spectrum has now been measured in a consistent fashion down to below $R \simeq 1 h^{-1}$ Mpc.

The shaded region at larger R shows the results deduced from clustering data (Peacock 1997). It is clear that an $\Omega = 1$ universe requires the power spectrum at small scales to be higher than would be expected on the basis of an extrapolation from the large-scale spectrum. Depending on assumptions about the scale dependence of bias, such a ‘feature’ in the linear spectrum may also be required in order to satisfy the small-scale present-day nonlinear galaxy clustering (Peacock 1997). Conversely, for low-density models, the empirical small-scale spectrum appears to match reasonably smoothly onto the large-scale data.

Figure 2 also compares the empirical data with various physical power spectra. A CDM model (using the transfer function of Bardeen *et al.* 1986) with shape parameter $\Gamma = \Omega h = 0.25$, is shown as a reference for all models. This appears to have approximately the correct shape, although it overpredicts the level of small-scale power somewhat in the low-density cases. A better empirical shape is given by MDM with $\Omega h \simeq 0.4$ and $\Omega_\nu \simeq 0.3$. However, this model only makes physical sense in a universe with high Ω , and so it is only shown as the lowest curve in figure 2c, reproduced from the fitting formula of Pogosyan & Starobinsky (1995; see also Ma 1996). This curve fails to supply the required small-scale power, by about a factor of three in σ ; lowering Ω_ν to 0.2 still leaves a very large discrepancy. This conclusion is in agreement with, for example, Mo & Miralda-Escudé (1994), Ma & Bertschinger (1994), Ma *et al.* (1997) and Gardner *et al.* (1997).

All the models in figure 2 assume $n = 1$; in fact, consistency with the COBE results for this choice of σ_8 and Ωh requires a significant tilt for flat low-density CDM models, $n \simeq 0.9$ (whereas open CDM models require n substantially above unity). Over the range of scales probed by LSS, changes in n are largely degenerate with changes in Ωh , but the small-scale power is more sensitive to tilt than to Ωh . Tilting the $\Omega = 1$ models is not attractive, since it increases the tendency for model predictions to lie below the data. However, a tilted low- Ω flat CDM model would agree moderately well with the data on all scales, with the exception of the ‘bump’ around $R \simeq 30 h^{-1}$ Mpc. Testing the reality of this feature will therefore be an important task for future generations of redshift survey.

(b) Collapse redshifts and ages for red radio galaxies

Are the collapse redshifts inferred above consistent with the age data on the red radio galaxies? First bear in mind that in a hierarchy some of the stars in a galaxy will inevitably form in subunits before the epoch of collapse. At the time of final collapse, the typical stellar age will be some fraction α of the age of the universe at that time:

$$\text{age} = t(z_{\text{obs}}) - t(z_c) + \alpha t(z_c). \quad (4.1)$$

We can rule out $\alpha = 1$ (i.e. all stars forming in small subunits just after the big bang). For present-day ellipticals, the tight colour-magnitude relation only allows an approximate doubling of the mass through mergers since the termination of star

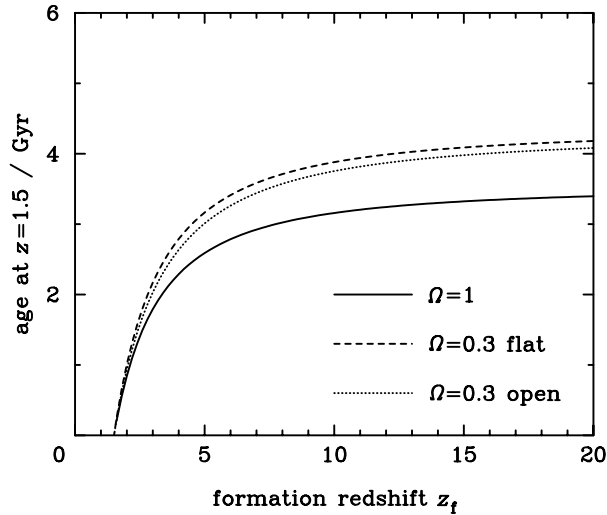


Figure 3. The age of a galaxy at $z = 1.5$, as a function of its collapse redshift (assuming an instantaneous burst of star formation). In all cases, the present age of the universe is forced to be 14 Gyr.

formation (Bower *et al.* 1992). This corresponds to $\alpha \simeq 0.3$ (Peacock 1991). A non-zero α just corresponds to scaling the collapse redshift as

$$\text{apparent}(1 + z_c) \propto (1 - \alpha)^{-2/3}, \quad (4.2)$$

since $t \propto (1 + z)^{-3/2}$ at high redshifts for all cosmologies. For example, a galaxy which collapsed at $z = 6$ would have an apparent age corresponding to a collapse redshift of 7.9 for $\alpha = 0.3$.

Converting the ages for the galaxies to an apparent collapse redshift depends on the cosmological model, but particularly on H_0 . Some of this uncertainty may be circumvented by fixing the age of the universe. After all, it is of no interest to ask about formation redshifts in a model with, for example, $\Omega = 1$, $h = 0.7$, when the whole universe then has an age of only 9.5 Gyr. If $\Omega = 1$ is to be tenable, then either $h < 0.5$ against all the evidence or there must be an error in the stellar evolution time-scale. If the stellar time-scales are wrong by a fixed factor, then these two possibilities are degenerate. It therefore makes sense to measure galaxy ages only in units of the age of the universe; or, equivalently, to choose freely an apparent Hubble constant that gives the universe an age comparable to that inferred for globular clusters. In this spirit, figure 3 gives apparent ages as a function of effective collapse redshift for models in which the age of the universe is forced to be 14 Gyr (see, for example, Jimenez *et al.* 1996).

This plot shows that the ages of the red radio galaxies are not permitted very much freedom. Formation redshifts in the range 6–8 predict an age of close to 3.0 Gyr for $\Omega = 1$, or 3.7 Gyr for low-density models, irrespective of whether Λ is non-zero. The age– z_c relation is rather flat, and this gives a robust estimate of age once we have some idea of z_c through the abundance arguments. It is therefore rather satisfying that the ages inferred from matching the rest-frame UV spectra of these galaxies are close to the above figures.

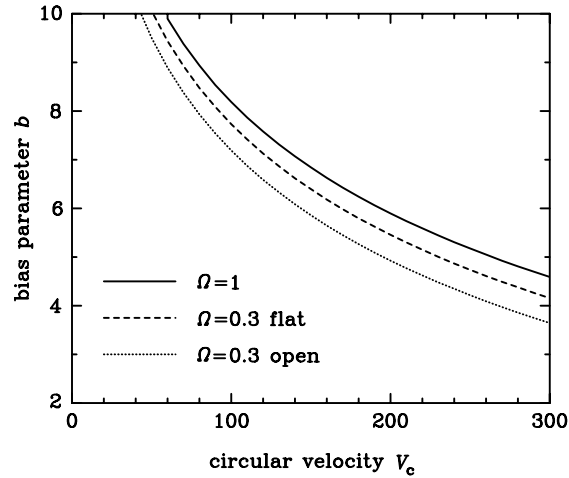


Figure 4. The bias parameter at $z = 3.2$ predicted for the Lyman-limit galaxies, as a function of their assumed circular velocity. A substantial bias in the region of $b \simeq 6$ is predicted rather robustly.

(c) *The global picture of galaxy formation*

It is interesting to note that it has been possible to construct a consistent picture that incorporates both the large numbers of star-forming galaxies at $z \lesssim 3$ and the existence of old systems which must have formed at very much larger redshifts. A recent conclusion from the numbers of Lyman-limit galaxies and the star-formation rates seen at $z \simeq 1$ has been that the global history of star formation peaked at $z \simeq 2$ (Madau *et al.* 1996). This leaves three possibilities open for the very old systems: they may be simply the rare precursors of this process, and form unusually early; they may be a relic of a second peak in activity at higher redshift, such as is commonly invoked for the origin of all spheroidal components; or dust obscuration may mean that the global star-formation rate has been underestimated. While a bimodal or dust-obscured history of star formation cannot be rejected, the rareness of the red radio galaxies indicates that there is no difficulty with the first picture. This can be demonstrated quantitatively by integrating the total amount of star formation at high redshift. According to Madau *et al.* (1996), the star-formation rate at $z = 4$ is

$$\dot{\rho}_* \simeq 10^{7.3} h M_\odot \text{Gyr}^{-1} \text{Mpc}^{-3}, \quad (4.3)$$

declining roughly as $(1+z)^{-4}$. This is probably an underestimate by a factor of at least three, as indicated by suggestions of dust in the Lyman-limit galaxies (Pettini *et al.* 1997), and by the prediction of Pei & Fall (1995), based on high- z element abundances. If we scale by a factor of three, and integrate to find the total density in stars produced at $z > 6$, this yields

$$\rho_*(z_f > 6) \simeq 10^{6.2} M_\odot \text{Mpc}^{-3}. \quad (4.4)$$

Since the red mJy galaxies have a density of $10^{-5.87} h^3 \text{Mpc}^{-3}$ and stellar masses of order $10^{11} M_\odot$, there is clearly no conflict with the idea that these galaxies are the first stellar systems of L^* -size which form en route to the general era of star and galaxy formation.

(d) Predictions for biased clustering at high redshifts

An interesting aspect of these results is that the level of power on 1 Mpc scales is only moderate: $\sigma(1 h^{-1} \text{ Mpc}) \simeq 2$. At $z \simeq 3$, the corresponding figure would have been much lower, making systems like the Lyman-limit galaxies rather rare. For Gaussian fluctuations, as assumed in the Press–Schechter analysis, such systems will be expected to display spatial correlations that are strongly biased with respect to the underlying mass. The linear bias parameter depends on the rareness of the fluctuation and the RMS of the underlying field as

$$b = 1 + \frac{\nu^2 - 1}{\nu\sigma} = 1 + \frac{\nu^2 - 1}{\delta_c} \quad (4.5)$$

(Kaiser 1984; Cole & Kaiser 1989; Mo & White 1996), where $\nu = \delta_c/\sigma$, and σ^2 is the fractional mass variance at the redshift of interest.

In this analysis, $\delta_c = 1.686$ is assumed. Variations of order 10% in this number have been suggested by authors who have studied the fit of the Press–Schechter model to numerical data. These changes would merely scale $b - 1$ by a small amount; the key parameter is ν , which is set entirely by the collapsed fraction. For the Lyman-limit galaxies, typical values of this parameter are $\nu \simeq 3$, and it is clear that very substantial values of bias are expected, as illustrated in figure 4.

This diagram shows how the predicted bias parameter varies with the assumed circular velocity, for a number density of galaxies fixed at the level observed by Steidel *et al.* (1996). The sensitivity to cosmological parameter is only moderate; at $V_c = 200 \text{ km s}^{-1}$, the predicted bias is $b \simeq 4.6, 5.5, 5.8$ for the open, flat and critical models, respectively. These numbers scale approximately as $V_c^{-0.4}$, and b is within 20% of 6 for most plausible parameter combinations. Strictly, the bias values determined here are upper limits, since the numbers of collapsed haloes of this circular velocity could in principle greatly exceed the numbers of observed Lyman-limit galaxies. However, the undercounting would have to be substantial: increasing the collapsed fraction by a factor of 10 reduces the implied bias by a factor of about 1.5. A substantial bias seems difficult to avoid, as has been pointed out in the context of CDM models by Baugh *et al.* (1998).

Comparing the bias values in figure 4 with those observed directly (§ 2*b*), we see that the observed value of b is quite close to the prediction in the case of $\Omega = 1$; suggesting that the simplest interpretation of these systems as collapsed rare peaks may well be roughly correct. Indeed, for high circular velocities there is a danger of exceeding the predictions, and it would create something of a difficulty for high-density models if a velocity as high as $V_c \simeq 300 \text{ km s}^{-1}$ were to be established as typical of the Lyman-limit galaxies. For low Ω , the ‘observed’ bias falls faster than the predictions, so there is less danger of conflict. For a circular velocity of 200 km s^{-1} , we would need to say that the collapsed fraction was underestimated by roughly a factor of 10 (i.e. increase the values of σ in figure 2 by a factor of *ca.* 1.5) in order to lower the predicted bias sufficiently, either by postulating that the conversion from velocity to R is systematically in error, or by suggesting that there may be many haloes that are not detected by the Lyman-limit search technique. It is hard to argue that either of these possibilities are completely ruled out. Nevertheless, we have reached the paradoxical conclusion that the observed large-amplitude clustering at $z = 3$ is more naturally understood in an $\Omega = 1$ model, whereas one might have expected the opposite conclusion.

5. Empirical predictions for CMB anisotropies

The recurring theme of this paper has been that it is now possible to measure the fluctuation spectrum empirically to an interesting precision. On large scales, this is possible using galaxy clustering to give the shape of the spectrum, with the cluster abundance giving the normalization. On small scales, we have seen how information on high-redshift galaxies gives answers that are reasonably consistent with extrapolation of the large-scale results. This situation is to be contrasted with the normal approach to measurements of CMB anisotropies, where the results are fitted by variants on CDM models, adjusting the parameters $(\Omega_m, \Omega_v, \Omega_B, h, n)$. If CMB data alone are considered, many combinations of these parameters can fit existing results; however, in many cases the predicted $z = 0$ matter fluctuation spectrum will be in gross disagreement with observation.

This problem is often tackled by requiring acceptable models to fit some statistic such as σ_8 . However, an alternative route is to recall that the CMB calculations are entirely linear, and that they are based on the evolution of a given Fourier mode from last scattering at $z \simeq 1100$ to the present. The equations involved are time symmetric, so there is no reason why the integration cannot be carried out backwards. If we believe that the amplitude of gravitational potential fluctuations at $z = 0$ has been measured as a function of scale, then it makes sense to place these fluctuations at last scattering and deduce an empirical prediction of the CMB fluctuations. In practice, this can be achieved by a process that resembles ‘designer inflation’: assume a suitable fluctuation spectrum at $z > 1100$ such that any features in the transfer function are cancelled, leaving the desired power spectrum at $z = 0$. Described in this way, the process sounds unnatural; however, the standard lore suggests that perturbations are generated at $z \sim 10^{28}$, so there is much more room at $z > 1100$ for unknown extra physics than there is at $z < 1100$.

This approach still leaves free the global cosmological parameters. The CMB results clearly depend on Ω_m and Ω_v , since the inferred fluctuation spectrum depends on these (although only weakly on Ω_v). The other parameters can be fixed at their empirical values, taken here to be $h = 0.65$ and $\Omega_B/\Omega = 0.1$. For an extreme empirical approach, no power would be assumed beyond the largest scale at which clustering is observed in the galaxy distribution ($k \simeq 0.02 h \text{ Mpc}^{-1}$). A reasonable alternative, adopted here, is to allow the spectrum to vary with some power-law index n on larger scales. Finally, the collisionless dark matter is taken to be cold and the fluctuations are assumed to be isentropic; variations of either of these assumptions would lead to larger fluctuations.

The results of this calculation are shown in figure 5, which was generated using a modified form of the CMBFAST code of Seljak & Zaldarriaga (1996). This looks quite different from the plots usually seen in this field, for the following reasons.

1. The normalization comes direct from the power spectrum; no attempt has been made to fit the CMB data.
2. The models are thus not COBE normalized, although they could be made to fit COBE by adjusting the large-wavelength index n . Open models would require $n > 1$, flat models $n < 1$.
3. Adjusting n in this way only affects C_ℓ for $\ell \lesssim 300$, since larger multipoles project to parts of k space probed by galaxy clustering.

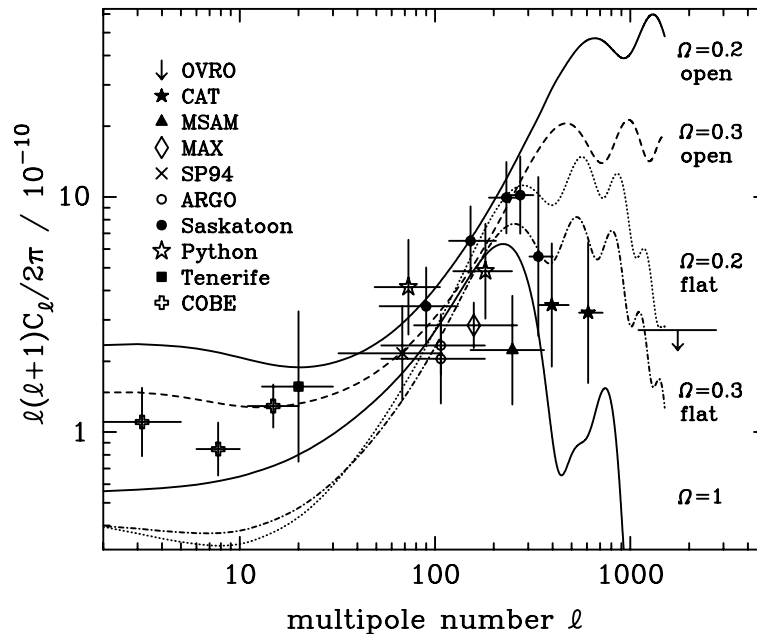


Figure 5. The angular power spectrum for the temperature fluctuations in the CMB. These predictions fix the matter power spectrum today to have the shape inferred from galaxy clustering and the normalization inferred from the abundance of rich clusters. The ‘observed’ values are adopted for the other cosmological parameters: $h = 0.65$, $\Omega_B/\Omega = 0.1$. At long wavelengths, where no galaxy clustering data exist, the spectrum is assumed to be scale invariant; failure to match COBE thus indicates that tilt is required. However, the power at $\ell \sim 2000$ is nearly spectrum independent, since this is where the normalization scale sits. The rejection of open models is thus very nearly model independent.

The last point is especially important, since it means that it is the high- ℓ clustering where robust predictions can be made. The $k \simeq 0.2 h \text{ Mpc}^{-1}$ waves that determine σ_8 project to $\ell \simeq 1200$ for $\Omega = 1$, or to $\ell \simeq 1200/\Omega$ and $1200/\Omega^{0.4}$ for open and flat models, respectively. This difference in angular-diameter distance is half of the reason why the predictions for open models in figure 5 are so much higher than the predictions for flat models with the same parameters. The other difference is the difference in linear growth suppression factors, which amounts roughly to a factor of $\Omega^{0.4}$ (equation (1.1)). Since the present-day power spectrum and its normalization is highly insensitive to Λ , there is thus a very simple recipe for predicting the CMB anisotropies for a given open model: calculate the corresponding flat case, boost the results by a factor of $\Omega^{-0.8}$ and translate the spectrum to higher ℓ by a factor of $\Omega^{-0.6}$. This recipe fails for the lowest multipoles, where spatial curvature is important. However, the critical $\ell \sim 1000$ results follow this scaling almost exactly.

The conclusion is, therefore, that for any flat model that roughly fits the CMB data, its open counterpart will be grossly in error, and this is just what is seen in practice. Flat models with $\Omega \gtrsim 0.3$ are acceptable, but open models are qualitatively wrong unless $\Omega \gtrsim 0.5$. It is interesting to note that this conclusion comes not so much from the modern data at $\ell \simeq 200$, but from the long-standing OVRO upper limit at $\ell \simeq 2000$. The inconsistency of this result with most open models was noted by Bond

& Efstathiou (1984), and all that has changed since then is that we now prefer to use the cluster normalization, rather than the unbiased normalization chosen by Bond & Efstathiou. This raises the amplitude for low- Ω models, making it that much harder for them to get anywhere near to the data.

6. Conclusions

The data on the abundances and clustering of both radio-loud and radio-quiet galaxies at high redshift appear to be in good quantitative agreement with the expectation of models in which structure formation proceeds through hierarchical merging of haloes of dark matter. Furthermore, the existing data yield an empirical measurement of the fluctuation spectrum on sub-Mpc scales. In general, this small-scale spectrum is close to what would be expected from an extrapolation of LSS measurements, but there are deviations in detail: $\Omega = 1$ places the small-scale data somewhat above the LSS extrapolation, whereas open low- Ω models suffer from the opposite problem; low- Ω Λ -dominated models fare somewhat better, especially with a slight tilt. These last models also account well for the $\ell \sim 1000$ CMB anisotropies if the dark matter is assumed to be pure CDM, normalized to COBE (whereas open models fail badly). Until recently, it appeared that geometrical tests such as the supernova Hubble diagram (Perlmutter *et al.* 1997) or gravitational lensing (Carroll *et al.* 1992; Kochanek 1996) were strongly inconsistent with Λ -dominated models, so the overall situation was badly confused. However, with recent developments in these areas now appearing to favour a non-zero Λ (Garnavich *et al.* 1998; Chiba & Yoshii 1997), it is possible that a consistent picture may be emerging.

The main remaining difficulty for Λ CDM lies in the shape of the large-scale power spectrum measured from the APM survey around $k = 0.1 h \text{ Mpc}^{-1}$. This is a region of the spectrum that is well within the capability of 2dF and Sloan, so we can confidently expect this problem to be either confirmed or removed within the next few years. The subject of structure formation thus stands at a critical point: either we are close to having a ‘standard model’ for galaxy formation and clustering, or we may have to accept that radical new ideas are needed. At the current rate of observational progress, the verdict should not be very far away.

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